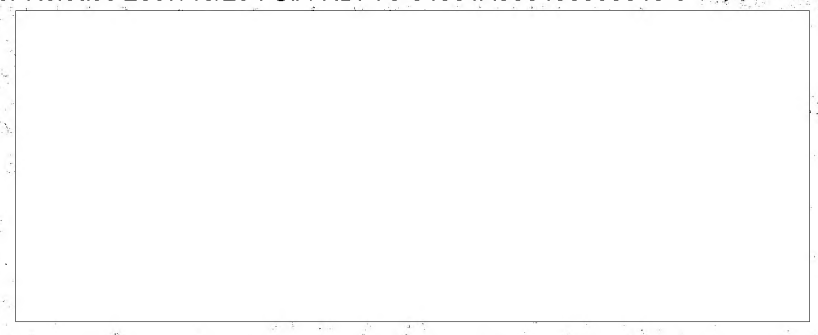

ON THE THEORY OF THE GERMANIUM RECTIFIER AND THE TRANSISTOR

H. Krömer

Zeits. für Phys. 134 (1953) 435 - 450
(From German)

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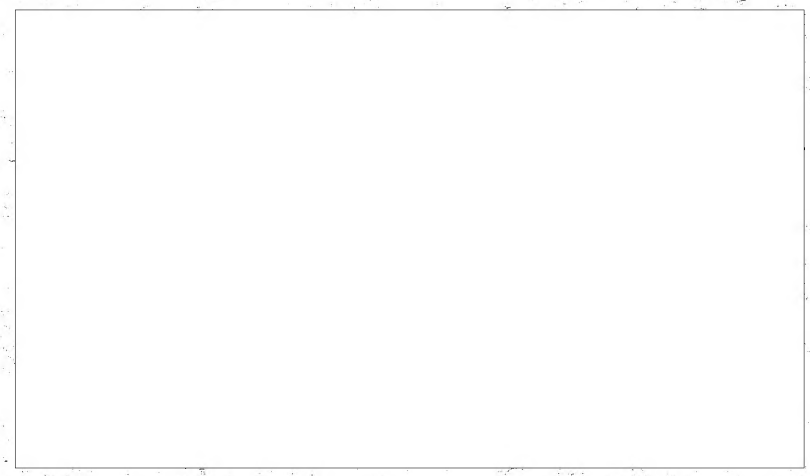

July, 1955

ON THE THEORY OF THE GERMANIUM RECTIFIER AND THE TRANSISTOR

H. Krömer

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In the case of the high field strengths prevailing in the boundary layers of semi-conductor-metal-contacts the transport of electrons and deficit electrons (holes) is not governed by the normal mobilities, which apply when the potential drop along a mean free path is small in relation to the thermal energy kT.

In the stronger fields there is a strong probability that the charge carriers will reach the interior of the band; in the limit case of very strong fields a particle oscillates back and forth in the band several times, between two collisions, and so progresses more slowly ("blocking effect"). Mobility then decreases so that the density of the particles rises in proportion to the field strength and the current density. The constants of proportionality (blocking constants) for holes are estimated at $2 \cdot 10^{3}$ watt^{-1}; it should be considerably smaller for the electrons.

The density of the blocked particles can reach the same order of magnitude as that of the impurity centres. The boundary layer potential then deviates considerably from the simple SCHOTTKY parabola form. Characteristic curves of Ge-transistors and diodes are calculated in accordance with this change in potential. If the parameters are suitably chosen high values are obtained for the current amplification factor in the case of transistors while in the case of diodes a displacement of the barrier characteristics has been observed.

I. INTRODUCTION

According to SCHOTTKY [13]* the level of the semi-conductor bands at the boundary layer between germanium and a metal, is raised by the amount of the so-called diffusion potential V_D when there is no applied voltage and by $dU + V_D$ with an applied voltage of U. A potential barrier appears which hinders electron motion; it is dependent upon U and its height above the Fermi level of the metal is

$$e\,\Phi_0 = V_D + E_L \qquad \ldots\ldots (1)$$

and is independent of U (Figure 1).

The image force attraction of the metal towards the electrons causes the peak of this barrier to be flattened by

$$\Delta V = \sqrt{\frac{e^3 F}{\eta}} \qquad \ldots\ldots (2)$$

(Figure 2), in which F stands for the field strength at the end of the boundary layer (boundary field strength). Instead of the static dielectric constant ϵ of the semi-conductor a "dynamic" dielectric constant $\eta < \epsilon$, is introduced here because when the electrons have a velocity of 10^7 to 10^8 cm/sec. and the distance of the maximum of the flattened barrier from the boundary surface

$$x_s = \pm\sqrt{\frac{e}{\eta F}}$$

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* For references, see end.

lies between some 10^{-7} and 10^{-6} cm., with a transit time of 10^{-13} to 10^{-15} sec., the static value for the dielectric constant $\epsilon = 16$ [5] can scarcely be considered as valid. The lattice polarisation phase also lags and still further reduces the effective dielectric constant.

In these circumstances equation (2) must be regarded as a first approximation and η as a parameter about which no theory can be put forward at present and which must therefore be determined experimentally. Measurements made by S. BENZER [3] on Ge-rectifiers give $\eta \approx 1$. W. OLDEKOP [10] reached the same findings with selenium rectifiers. We shall therefore accept this value in the following calculations and put $\eta = 1$.

Taking the image force into account the total potential barrier to be overcome by the electrons becomes

$$V_S = e\,\Phi_0 - \Delta V = V_D + E_L - \sqrt{\frac{e^3 F}{\eta}} \quad . \qquad \ldots\ldots (3)$$

In accordance with the diode theory of the crystal rectifier [4] [18] the value of the density of the electron stream flowing in the barrier direction for $eU \gg kT$ is

$$j_n = A\,T^2\,e^{-V_S/kT} \quad \text{where} \quad A = \frac{4\pi e m k^2}{h^3} \quad . \qquad \ldots\ldots (4)$$

In the case of a simple depletion boundary layer with N_0 as the concentration of impurity centres SCHOTTKY [13] gives the following

$$F = \sqrt{\frac{8\pi N_0}{\epsilon}\,(eU + V_D)} \quad . \qquad \ldots\ldots (5)$$

From (3) and (4) it follows, for $eU \gg V_D$

$$j_n = A\,T^2 \exp \frac{1}{kT}\left[-e\,\Phi_0 + \sqrt[4]{\frac{8\pi e^7 N_0}{\epsilon\,\eta^2}}\,\sqrt[4]{U}\right] .$$

Such rectilinear characteristics in the $(\log j/j_0,\ U)$-diagram were found by SEILER [12] for example in Si-detectors. N_0/η^2 can be subsequently determined from the slope of the rectilinear characteristics. SEILER, who took $\eta = \epsilon$ found values for N_0 which exceeded those otherwise determined by up to a tenth power. This confirms the supposition that $\eta < \epsilon$.

In the derivation of (5) it was assumed that the density of the charge carriers responsible for current transport, which in the interior of the semi-conductor is in conformity with that of the ionised impurity centres, falls to a very low value in the course of a short distance in the direction of the barrier layer, so that in the barrier layer only the charge density of the impurity centres has to be taken into account. As will be shown later, this assumption is only correct when the applied voltages are not too great. However, if the barrier field strength is great enough, the effective carrier mobility in the barrier layer is reduced so much that the carrier density increases sharply and may finally reach an order of magnitude equal to that of the impurity centres.

The calculation of the depletion boundary layer will be repeated with the space charge density as altered by this condition and it will then be applied to calculations of diode and transistor curves.

II. THE BLOCKING EFFECT

II,1. LAWS OF TRANSPORT IN WEAK AND STRONG FIELDS

The laws of transport for charge carriers depend on the extent to which the potential energy V of the electrons changes along a mean free path λ. The latter is obtained from the mobility b in accordance with

$$\lambda = \frac{\sqrt{2\pi m k T}}{e} \cdot b \quad \text{(see e.g. W. SCHOCKLEY [15], page 277).}$$

It follows from this that at room temperature and when

$$b_n = 3600 \text{ cm}^2 \text{ V}^{-1} \text{ sec}^{-1}, \qquad b_p = 1700 \text{ cm}^2 \text{ V}^{-1} \text{ sec}^{-1} \text{ [11]:}$$

$$\lambda_n = 2.5 \cdot 10^{-5} \text{ cm}, \qquad \lambda_p = 1.2 \cdot 10^{-5} \text{ cm}.$$

If $|\text{grad } V| = eF \ll \frac{kT}{\lambda}$ the following are true:

Density of electron current $\vec{j}_n = b_n [n \cdot \text{grad } V + kT \cdot \text{grad } n]$, (6a)

" hole " $\vec{j}_p = b_p [p \cdot \text{grad } V - kT \cdot \text{grad } p]$ (6b)

(n and p = electron and hole density).

The charge densities then become

$$\rho_n = -\frac{1}{b_n F} \cdot |\vec{j}_n - b_n kT \, \text{grad} n| ; \rho_p = +\frac{1}{b_p F} |\vec{j}_p + b_p kT \, \text{grad} p| . \quad (7)$$

If the current remains constant then the charge densities decrease with an increasing field.

When $eF \gtrsim \frac{kT}{\lambda}$ the ratios become very complex and it is only when the limit case of very strong fields is reached that simple laws are again obtained. If

$$\text{grad } V \geq eF_s = \frac{2B}{\lambda} , \quad \ldots\ldots (8)$$

where B represents the width of the valence or conduction band, the charge carriers in the field are no longer transported away but, before the next collision, they oscillate between the edges of the band which is inclined in the field; thus they remain longer in the same place with the result that the density of the particles again rises.

We have no experimental data about the widths B_V and B_L of the valence and conduction bands for Ge and as far as theoretical investigations are concerned we have only an approximate calculation by H. MÜLLER [9] from a simplified WIGNER-SEITZ process of which the quantitative usefulness is somewhat doubtful.

It seems certain, however, that the conduction band is considerably wider than the forbidden band and overlaps with several higher bands, while the valence band has a width of the same order of magnitude as the forbidden band. MÜLLER has found $B_V < E_B$ but the E_B is too great of course. We should like to assume that B_V is of the same order of magnitude of about 1 eV, but B_L is $\gg 1$ eV. When $B_V = 1$ eV and $\lambda = 1.2 \cdot 10^{-5}$ cm (8) gives a value for $F \geq 1.7 \cdot 10^5$ cm^{-1}, an easily attainable value. We are investigating this case in greater detail.

II, 2. CALCULATION OF THE BLOCKING CONSTANTS

It is obvious that the space charge density of the blocked particles has the value

$$\rho_s = \pm \frac{j.e.F}{(\Delta E/\tau)} \quad \text{for} \left\{ \begin{matrix} \text{holes} \\ \text{electrons} \end{matrix} \right\} .$$

ΔE is the energy loss for each collision and τ is the time between two collisions. Since ΔE and τ are statistically independent the following applies:

$$\rho_s = \pm \frac{j.e.F}{\overline{\Delta E} \cdot \overline{(1/\tau)}} . \quad \ldots\ldots (9)$$

$\overline{\Delta E}$ and $\overline{(1/\tau)}$ can be estimated from the theory of electron scattering at lattice waves.

II, 2a

When an electron or a hole is scattered at a lattice wave of frequency ν, it changes its energy by $\pm h\nu$. The probability of energy loss in proportion to that for energy gain is $N + 1/N$, where N is the mean quantum number to which the lattice wave is excited. With $2N + 1$ collisions there is therefore an energy loss of $h\nu$, i.e. an average loss per ν-collision

$$\Delta E_\nu = \frac{h\nu}{2N + 1} . \quad \ldots\ldots (10)$$

The probabilities of scattering at two waves of varying frequencies ν_1 and ν_2 are approximately in the proportion

$$\frac{W_1}{W_2} = \frac{h\nu_1 (2N_1 + 1)}{h\nu_2 (2N_2 + 1)} \qquad \ldots\ldots (11)$$

(Equations 10 and 11 follow from equations 34,33 and 34,36 in SOMMERFELD and BETHE [17] where $C_1 = C_2$ and $q_1 : q_2 = \nu_1 : \nu_2$).

If we introduce the number $z(\nu)$ of the lattice vibrations per frequency interval 1, then, from (10) and (11)

$$\overline{\Delta E} = \frac{\int \Delta E_\nu \cdot h\nu (2N_\nu + 1)\, z(\nu)\, d\nu}{\int h\nu (2N_\nu + 1)\, z(\nu)\, d\nu} = \frac{\int (h\nu)^2 z(\nu)\, d\nu}{\int h\nu \cdot (2N_\nu + 1)\, z(\nu)\, d\nu} . \qquad (12)$$

Substituting for $z(\nu)$ the DEBYE spectrum

$$z(\nu) = \begin{cases} \text{const.} \cdot \nu^2 & \text{for } \nu \leqslant \nu_g = \dfrac{k\Theta}{h} \\ 0 \end{cases}$$

(Θ DEBYE-temperature). Restricting temperature to $T > \Theta$ gives

$$2N_\nu + 1 = 1 + \frac{2}{e^{h\nu/kT} - 1} \approx 2\frac{kT}{h\nu} . \qquad \ldots\ldots (13)$$

From which we get

$$\overline{\Delta E} = \frac{3}{10}\frac{(k\Theta)^2}{kT} \qquad \ldots\ldots (14)$$

II, 2b

If equation (8) is fulfilled, the electron or hole moves, on the average, backwards and forwards through the whole band before it is subjected to the next collision. In relation to the collision area this means that it moves uniformly straight through the whole BRILLOUIN-zone corresponding to this band. Since the quantum conditions in the collision area are distributed with constant density, it remains an equal time in each state and the collision probability $(\overline{1/\tau})$ is obtained simply as the mean value over the whole BRILLOUIN-zone. Introducing the free path λ and the group velocity $\nu = \text{grad}_p E(P)$ we get:

$$\left(\overline{\frac{1}{\tau}}\right) = \frac{1}{V_P}\iiint \frac{|\text{grad}_p E(P)|}{\lambda(P)}\, dP_x\, dP_y\, dP_z . \qquad \ldots\ldots (15)$$

For further evaluation it is necessary to make the serious assumption that λ is independent of the velocity and hence of P, not only at the edge of the band but also in the interior. Then we have

$$\left(\overline{\frac{1}{\tau}}\right) = \frac{1}{\lambda}\,|\text{grad}_p E| . \qquad \ldots\ldots (16)$$

Since we do not know the exact form of the surfaces of constant energy in the P-area, we are unable to compute this mean value exactly. We can obtain at least the correct order of magnitude, however, by substituting for the gradients in (16) the magnitude

$$\frac{\text{Energy difference inside the cell}}{\text{Half the length of cell edge}} = \frac{B}{h/2a}$$

(a = lattice constant). Thus we get

$$\left(\overline{\frac{1}{\tau}}\right) = \frac{a}{h} \cdot \frac{2B}{\lambda} \cdot g , \qquad \ldots\ldots (17)$$

where $\mathcal{G}$ is a numerical factor dependent on "cell geometry" and the form of the energy surfaces and is of the order of magnitude unity. We shall continue to use $\mathcal{G} = 1$; this value applies exactly, e.g. for unidimensional lattices.

II, 2c

If (14) and (17) are inserted in (9) we get

$$\rho = j \cdot e \cdot F \cdot \frac{10}{3} \cdot \frac{h}{a} \cdot \frac{kT}{(k\Theta)^2} \cdot \frac{\lambda}{2B} = j \cdot e F \cdot s, \qquad (18)$$

in which the "blocking constant" S is defined by

$$s = \frac{1}{\Delta E \cdot (1/\tau)} = \frac{10}{3} \cdot \frac{h}{a} \cdot \frac{kT}{(k\Theta)^2} \cdot \frac{1}{e F_S} \qquad \ldots\ldots (19)$$

The values to be substituted in the case of Ge for deficit electrons ($B_V = 1$ eV, $\lambda_p = 1.2 \cdot 10^{-5}$ cm; $\Theta = 400°$ K[1], $a = 5.6 \cdot 10^{-8}$ cm) give, at room temperature

$$F_{S,p} = 1.7 \cdot 10^5 \text{ V cm}^{-1}$$

$$s_p = 2 \cdot 10^8 \frac{\text{cm}^{-3}}{\text{V cm}^{-1} \text{ Amp cm}^{-2}} = 2 \cdot 10^8 \text{ W}^{-1}. \qquad \ldots\ldots (20)$$

The usual point contacts have diameters of about 10^{-3} cm and the hole currents flowing in the transistor are of the order of 1 mA. This gives current densities of some hundred amp . cm^{-2}. As the field strengths which occur lie around 10^5 V . cm^{-1}, there arise blocking concentrations of the same order as those of the impurity centres.

II, 3 DISCUSSION OF THE OMISSIONS AND APPROXIMATIONS MADE

(a) Equation (11) cannot be applied strictly because the BLOCH interaction constant C^* which is part of the collision probability is not quite independent of the starting and finishing states. But as in (12) it appears in both the numerator and the denominator it has only a small influence. If, for example, W were proportional to $(h\nu)^{1+\alpha} \cdot (2N + 1)$ with $\alpha \neq 0$, then in (14) there would be only one factor

$$\left(1 + \frac{\alpha}{3}\right) \Big/ \left(1 + \frac{\alpha}{5}\right)$$

on the right hand side which is of the order of magnitude of unity.

(b) For the same reason the use of the simple DEBYE spectrum is only of small significance.

(c) The blocking constant S does not depend on temperature. The integral in the denominator in (12) is, apart from constants, precisely the total collision probability, and so proportional to $(1/\tau)$, with the result that the dependence on T disappears. The approximation (13) merely simplified the calculation (λ proportional to $1/T$ for $T > \Theta$); the numerical value (20) if it applies at all, applies to all temperatures.

(d) Problematic in (18) is the assumption of a mean free path which is independent of velocity and direction. This is correct at the edge of the band but doubtful in the interior. ([15] Ch. 17). There is as yet no exact theory about this case and nothing can be put forward here. There is therefore the possibility that what has been said may be corrected in the future.

II, 4 FIELDS OF MEDIUM STRENGTH

In fields of medium strength ($kT/\lambda < |\text{grad } V| < e F_S$) there is no longer the same degree of probability for every part of the collision area. As the field decreases, the particles withdraw more and more to the areas of least energy, i.e. to the centre of the P-area which correspond, for holes, to the upper edge of the band and for electrons, to the lower edge. There is therefore in (15) a factor in the integral, decreasing towards the edges, and of which the exact form cannot be given at present.

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* The values given vary a great deal. We associate ourselves with HILL and PARKINSON [7].

It is also difficult to make a calculation for $\overline{\Delta E}$, because there is no further possibility of energy loss at the edge of the band.

In the case of weak fields ($e F \ll k T/\lambda$) (7) must be omitted. In the region of medium field strength we have to fall back on a plausible interpolation between (7) and (18). The simplest possibility seems to be the sum of both expressions since each part in the "false" region tends towards zero. ♦ When both kinds of carrier are taken into consideration the total space charge is

$$\rho = e N_0 + \left[\frac{j_p + b_p \cdot k T \cdot \operatorname{grad} p}{b_p} - \frac{j_n - b_n \cdot k T \cdot \operatorname{grad} n}{b_n}\right] \cdot \frac{1}{F} + \left.\begin{array}{r} \\ + e (s_p j_p - s_n j_n) \cdot F . \end{array}\right\} \quad \ldots\ldots (21)$$

Only s_p had been estimated; owing to the greater bandwidth s_n should be considerably larger. In later examples we shall use the arbitrary value $s_n = 0.2 \cdot s_p$.

II,5 CALCULATION OF BARRIER FIELD STRENGTHS

II,5a Neglecting the blocking term in (21) leads to the normal depletion barrier layer of which SCHOTTKY [13] has shown that the true space charge character can be replaced by a constant charge density $\rho = e \cdot N_0$ of finite thickness l.

Hence, the calculation remains the same for a limited thickness of the barrier layer, but for the charge density we insert

$$\rho = e \cdot N_0 + e s_p \cdot j_s \cdot F, \quad \ldots\ldots (21a)$$

where

$$j_s = j_p - \frac{s_n}{s_p} \cdot j_n . \quad \ldots\ldots (22)$$

The POISSON-equation is then, in the case of surface contacts:

$$\frac{d^2\phi}{dx^2} = -\frac{4\pi e}{\epsilon}\left[N_0 + s_p \cdot j_s \cdot \frac{d\phi}{dx}\right]$$

with the limiting conditions

$$e [\phi (l) - \phi (0)] = e U + V_D; \quad \phi'(l) = 0.$$

By eliminating l we find the implicit law for the boundary field strength F_R:

$$e U + V_D = \frac{\epsilon}{4\pi} \cdot \frac{N_0}{(s_p \cdot j_s)^2}\left[\frac{s_p \cdot j_s \cdot F_R}{N_0} - \ln\left(1 + \frac{s_p \cdot j_s \cdot F_R}{N_0}\right)\right] \quad (23)$$

II,5b The contacts which are of greater interest to us are the point contacts with a diameter of about 10^{-3} cm. We will therefore go on to calculate F_R for spherical contacts (radius r_0) with a spherical surface layer (radius R).

Because $j(r)$ is no longer constant spatially but

$$j(r) = j \cdot \frac{r_0^2}{r^2} *$$

the POISSON-equation now becomes

$$\frac{d^2\phi}{dr^2} + \frac{2}{r} \cdot \frac{d\phi}{dr} = -\frac{4\pi e}{\epsilon}\left[N_0 + s_p \cdot j_s \cdot r_0^2 \cdot \frac{1}{r^2}\frac{d\phi}{dr}\right]$$

with the limiting conditions

$$e [\phi (R) - \phi (r_0)] = e U + V_D; \quad \phi'(R) = 0.$$

---

* When j is given without argument, it will be intended in future to mean in every case the current density in the contact surface.

The relation between U, F_R and R will then be given by the pair of equations

$$F_R = \frac{3\alpha}{r_0^2} \cdot e^{\beta/r_0} \cdot \int_{r_0}^{R} e^{-\beta/s} \cdot s^2 \, ds, \qquad \ldots\ldots (24a)$$

$$e\,U + V_D = \frac{r_0^2}{\beta} \cdot e\,F_R - \frac{e\,\alpha}{\beta}(R^3 - r_0^3) \qquad \ldots\ldots (24b)$$

where

$$\alpha = \frac{4\pi e}{\epsilon} \cdot \frac{N_0}{3} \quad \text{and} \quad \beta = \frac{4\pi e}{\epsilon} \cdot s_p \cdot j_s \cdot r_0^2 \, .$$

Figure 5 shows $F_R\,(U,\, j_s)$ in which $s_p = 2 \cdot 10^8$ w^{-1} and $N_0 = 10^{15}$ cm^{-3} was substituted, for a point contact in which $r_0 = 10^{-3}$ cm. For purposes of comparison there are also some (dotted) curves representing surface contacts.

III. APPLICATION TO THE CHARACTERISTIC CURVES

By combining equations (3) and (4) and Figure 5 it is possible to determine the family of curves having j_s as the parameter.

III,1 TRANSISTOR CURVES

III,1a In the transistor practically the total number of holes from j comes from the emitter current and the curves in which we are interested are those with definite j_p. These are obtained by plotting the curves with j_s as parameter and the family of horizontal straight lines

$$j_n = \frac{s_p}{s_n}(j_p - j_s)$$

on the $(j_n - U)$ diagram and joining the points of intersection of correlated curves (Figure 6). The curves for the total current $j_n + j_p$ are obtained simply, by vertical displacement about j_p. Assuming that the emitter current is simply hole current which goes entirely to the collector, we have the characteristic curves for the transistor.

With increasing voltage the hole current increases the electron current, but this continues only until the increase in the blocking charge of the latter has once more compensated for that of the hole current. The curves corresponding to various values of j_p will then become almost parallel. The "internal current amplification"

$$\alpha_i = \left(\frac{\partial\,(j_n + j_p)}{\partial\, j_p}\right)_{U\,=\,\text{const.}}$$

of the contact thus increases at first with the voltage but finally remains almost constant at

$$\alpha_i < 1 + \frac{s_p}{s_n} \, .$$

The rate of increase of α depends largely on Φ_0 and T and it can generally be said that the higher the open circuit current of the contact, the faster will α increase.

Figure 7a and b, supplemented by Figure 8 show two transistor curves with different values of Φ_0. The data are $N_0 = 10^{15}$ cm^{-3}, $r_0 = 10^{-3}$ cm, $s_p : s_n = 5$, $T = 300°$ K and $e\,\Phi_0 = 0.38$ and 0.50 eV.

III,1b While, in the sphere of validity of the diffusion theory the only valid values for α_i are

$$\alpha_i < 1 + \frac{b_n}{b_p} \approx 3 \quad [14],$$

the blocking effect creates the possibility of higher values with a maximum of

$$1 + \frac{s_p}{s_n}, \text{ provided that } \frac{s_p}{s_n} > \frac{b_n}{b_p} \, ,$$

which we feel inclined to regard as highly probable. The general shape of the curves is also satisfactorily reproduced by Figure 7, even when the collectors in question are not too highly formed.*

The forming process brings about marked changes in the curves: The α-values rise and simultaneously become independent of the voltage, down to far lower voltages than were previously obtainable, so that the family of characteristic curves becomes practically parallel above the order of 1 V. The explanation is not yet known. It could be supposed that there is an increase in F_R due to incoming donors and also a drop in Φ_0 due to some boundary layer effect [2]. It is open to question, however, whether this explanation is adequate. We think it highly probable that there will be further changes in the boundary layer and that these will modify the field-current dependence given by the diode theory (equations 3 and 4). Another deviation from the diode theory is represented by the p-n-Hook theory (SCHOCKLEY [14], [15], [16]) but this does not seem to be without its difficulties.

III, 2 DIODE CHARACTERISTIC CURVES

III, 2a In the case of diodes there is no external source of holes available and the hole current consists of the small number of holes which already exist in the semi-conductor or which move in from the surface or the base electrode. These holes reach the collector by diffusion and by the potential drop created in the semi-conductor by the electron current from the collector. If the electron current is sufficiently high (to which case we will confine ourselves), the diffusion can be ignored and the hole current becomes proportional to the electron current

$$j_D = G \cdot j_n \cdot \qquad \ldots\ldots (25)$$

When the temperature is evenly distributed and if the surface conductivity can be neglected we have

$$G = G_0 = \frac{b_p \cdot p_0}{b_n \cdot n_0} = \frac{b_p}{b_n} \cdot e^{-\frac{E_V - E_L}{kT}} .$$

In general G deviates from this value for three reasons:

1. The temperature is not uniform owing to contact heating.
2. There is conductivity at the surface.
3. If the base electrode is not completely barrier-free it may happen that $j_p/j_n > G_0$ at the base in which case the latter acts as an emitter.

We shall not attempt any general assessment of G but will investigate the form of the characteristic in relation to G.

The curves are obtained from the family of curves for a fixed j_S by intersection with the family of straight lines

$$j_n = \frac{j_S}{G - \frac{s_n}{s_p}}$$

and joining the corresponding points of intersection.

III, 2b Where the contacts have a high barrier resistance the current is so low that the blocking effect only becomes noticeable at high voltages. Thus in the region of moderate voltages it is only necessary to take into account the curves of $j_S = 0$. This has been done in Figure 8 for $e\,\Phi_0 = 0.54$ eV, $V_0 = 10^{-3}$ cm and $T = 298°K$ and the result compared with the curve of the same temperature by BENZER [3], (Figure 10). In these calculations the values $N_0 = 10^{15}$ cm^{-3} and $\eta = 1$ were used. The agreement is satisfactory. It would be substantially worse with a higher value of η, a higher value of N_0, which would allow a higher value for η also is improbable. The purpose of this comparison is to show the order of magnitude of Φ_0 for high barrier resistance rectifiers and to demonstrate that an explanation of the experiments depends on $\eta \approx 1$.

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* See [1], Figure 3 or [15], Figures 2 and 6.

III,2c Where $G < s_n/s_p$ the blocking effect of the holes is weaker than that of the electrons. We are only concerned with intersection points with negative j_s; the current is less than for $j_s = 0$. There is nothing special to note.

Now let $G > s_n/s_p$. Because of (22) it follows from $j_n = j_n(U, j_s)$

$$dj_n = \left(\frac{\partial j_n}{\partial U}\right)_{j_s} \cdot dU + \left(\frac{\partial j_n}{\partial j_s}\right)_U \cdot \left(dj_p - \frac{s_n}{s_p} \cdot dj_n\right) .$$

If $dj = dj_n + dj_p$ and $dj_n = \frac{dj}{1+G}$; $dj_p = \frac{G \cdot dj}{1+G}$ it follows then that

$$\frac{dj}{dU} = \left(\frac{\partial j_n}{\partial U}\right)_{j_s} \cdot \frac{1+G}{1 - \left(G - \frac{s_n}{s_p}\right) \cdot \left(\frac{\partial j_n}{\partial j_s}\right)_U} \qquad \ldots\ldots (26)$$

The disappearance of the denominator causes the curve to bend round into an area of negative differential resistance. Since $\partial j_n/\partial j_s$ rises without limit as U and j_s increase, this is ultimately the case for all curves in which $G > s_n/s_p$. An example is shown in Figure 9.

We do not think it impossible for G to become sufficiently large, owing to heating of the contact and surface conductivity, and for this mechanism to be responsible in many cases for the bending of the curves which has been observed.

III,3 THERMAL EFFECTS

III,3a The curves which have been calculated so far have been isothermic, i.e. related to a constant contact temperature. In actual fact, however, the contacts become heated approximately in proportion to the transposed Joule heat:

$$T_K - T_O = \delta \cdot (j \cdot U). \qquad \ldots\ldots (27)$$

The following applies to hemispherical contacts in an area of semi-conductor which is not too small:

$$\delta = \frac{r_O}{x} \approx 1.7 \cdot 10^{-3} {}^\circ\mathrm{C} \cdot \mathrm{W}^{-1}\ \mathrm{cm}^2$$

or

$$\frac{\delta}{2\pi r_O^2} = \frac{1}{2\pi x\, r_O} \approx 270 {}^\circ\mathrm{C}\ \mathrm{W}^{-1}.$$

It is assumed here that $r_O = 10^{-3}$ cm and the heat conductivity x is 0.14 cal sec^{-1} cm^{-1} $^\circ\mathrm{C}^{-1}$ [6].

If the current is purely electron current T_K must be inserted in place of T_O, but since T_K itself is dependent on j and U the direct current curves are obtained by intersection of the family of curves relating to different values of T_K (which are isothermic), with those of the hyperbolas

$$j = \frac{T_K - T_O}{\delta \cdot U}$$

and by joining up the corresponding points of intersection [8] (This is done best in practice with the log-log scale). Here, even though the isotherms continue further, there is generally a bending of the curves. (Figure 10).

If there is a noticeable hole current this simple form of representation is no longer possible.

III,3b The explanation of the bending of the curves by purely thermal effects has previously been mentioned by HUNTER [8]. This simple explanation is opposed by the fact that, even at 30 mc/s, the high frequency curve exhibits bending with a limit-voltage of the same order as with direct current ([18], p.381), although at these frequencies temperature change is no longer a factor. Hence, there must be some mechanism which causes even the isothermic curves to bend. It was seen above that such a mechanism was provided by the blocking effect.

III,3c The dependence of the bending region upon temperature cannot be said to have been correctly represented by the theory just offered, which is in fact an extension of the diode theory based on the blocking effect. According to BENZER, the product of current and voltage at the bending point shows a linear decline over a wide region as the base temperature rises and on extrapolation of the linear section, would disappear at about the setting in of self-conduction. However, the theory in its present form gives only a very slight dependence of this product on T_0 unless we make the implausible assumption that G drops severely as the temperature rises. Closer investigation shows that, in order to explain the experiments in the region of fairly high voltages, a much slighter dependence on T must be demanded of the isothermic curves than is given in the diode theory (and also the diffusion theory).

Whether in order to fulfil this demand it would be permissible to make the assumption that

$$\frac{d\,\Phi_0}{dT} > 0\,,$$

would require further study. We would prefer to assume that this discrepancy is a second indication that the surface layer model of the diode theory will have to be substantially improved.

REFERENCES

[1] J. BARDEEN and W.H. BRATTAIN. Phys. Rev. 75 (1949) 1208.

[2] J. BARDEEN and W.G. PFANN. Phys. Rev. 77 (1950) 401.

[3] S. BENZER. J. Appl. Phys. 20 (1949) 804.

[4] H.A. BETHE. NDRC-Rep. 14 23rd Nov. 1942, H.A. BETHE and R.G. SACHS. NDRC-Rep. 14, 10th Sept. 1942 and 15th June 1943.

[5] H.B. BRIGGS. Phys. Rev. 77 (1950) 284.

[6] A. GRIECO and H.C. MONTGOMERY. Phys. Rev. 86 (1952) 574.

[7] R.W. HILL and D.H. PARKINSON. Phil. Mag. 43 (1952) 309.

[8] L.P. HUNTER. Phys. Rev. 81 (1951) 151.

[9] H. MÜLLER. Ann. Phys. (6) 9 (1951) 141.

[10] W. OLDEKOP. Diss. Göttingen 1952.

[11] G.L. PEARSON, J.R. HAYNES and W. SHOCKLEY. Phys. Rev. 78 (1950) 295.

[12] K. SEILER. Naturforsch. 5a, (1950) 393.

[13] W. SCHOTTKY. Z. Physik 118 (1942) 539.

[14] W. SHOCKLEY. Phys. Rev. 78 (1950) 294.

[15] W. SHOCKLEY. Electrons and Holes in Semi-conductors, New York 1950.

[16] W. SHOCKLEY, M. SPARKS and G.K. TEAL. Phys. Rev. 83 (1951) 151.

[17] A. SOMMERFELD and H.A. BETHE. Handbuch der Physik, Bd.24/II, Kap.3.

[18] H.C. TORREY and C.A. WHITMER. Crystal Rectifiers, New York 1948.

E.L.

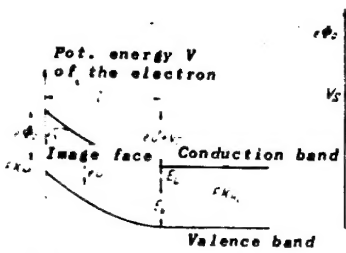

Fig.1: Boundary layer potential. E_L and E_V are the distances of the conduction and valence band of the semi-conductor from the Fermi level.

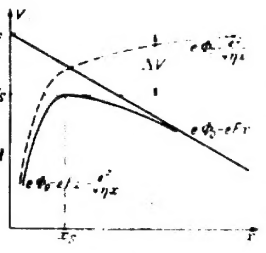

Fig.2: Disintegration of the potential barrier by the image force.

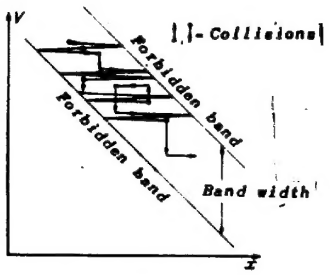

Fig.3: Electron motion in very strong fields.

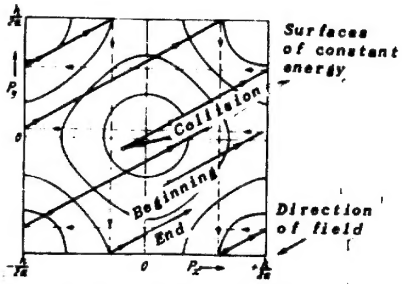

Fig.4: Section through the reduced collision space of a cubic crystal (a lattice constant). Motion of an electron in the outer field.

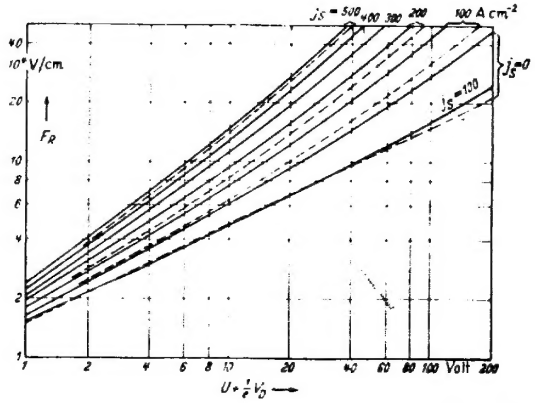

Fig.5: Boundary field strength for $N_o = 10^{15}$ cm^{-3} for point contacts with $r_o = 10^{-3}$ cm (———) and for surface contacts (- - - -).

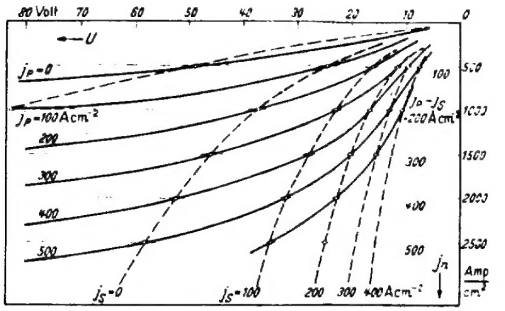

Fig.6: Construction of the electron current curves for constant hole current. Data: $r_o = 10^{-3}$ cm; $e\Phi_o = 0.38$ eV; $s_n : s_p = 1:5$; $T = 300°K$. Otherwise as in Figure 5.

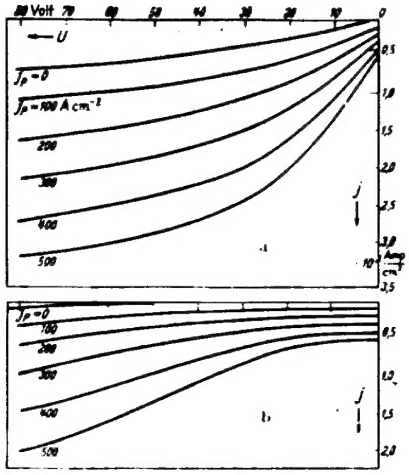

Fig.7a and b: Transistor curves for a) e Φ_o = 0.38 eV;
b) e Φ_o = 0.50 eV.
Other data as in Figure 6.

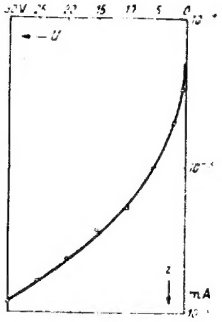

Fig.8: Comparison with a curve measured by BENZER.

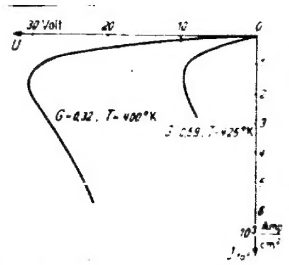

Fig.9: Curves showing bending. Data as in Figure 7a except temperature.

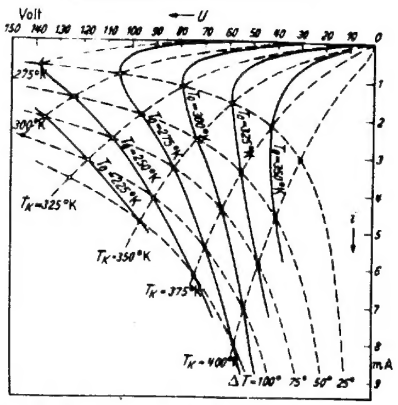

Fig.10: Construction of direct current curves. Data: e Φ_o = 0.50 eV; $r_s = 10^{-3}$ cm; j_s = 0; $\delta = 1.7 \cdot 10^{-3}$ °C cm^2 W^{-1}.